

N70-16342

NASA TECHNICAL TRANSLATION

NASA TT F-12,767

NASA TT F-12,767

PARAMETERS OF A HYDROGEN PLASMA IN
A SUPERSONIC SPHERICAL SOURCE

V.M. GOL'FARB, I.YE. KOSTYGOVA,
AND G.A. LUK'YANOV

Translation of: "Parametry vodorodnoy plazmy v
sverkhzvukovom sfericheskom istochnike"
Teplofizika Vysokikh Temperatur,
Vol. 7, No. 4, pp. 610-614, 1969

**CASE FILE
COPY**

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 JANUARY 1970

PARAMETERS OF A HYDROGEN PLASMA IN
A SUPERSONIC SPHERICAL SOURCEV.M. Gol'dfarb[†], I.Ye. Kostygova[†]
and G.A. Luk'yanov[†]

ABSTRACT: *The results of a numerical integration of a system of equations for a stationary source of nonequilibrium hydrogen plasma are given. Collision-radiation recombination is considered to be the governing elementary process. To clarify the effect of resorption, a calculation was performed for the cases of optically thin and optically thick plasma for all forms of radiation, and the optically thick plasma only for resonance radiation. In the flow region, considerable disturbance of thermal and ionization equilibrium was observed. Resorption of radiation leads to a considerable deviation of the electron temperature from the temperature of the heavy particles and a decrease in the recombination rate. In a quasi-stationary approximation, the populations of the hydrogen levels were determined on the basis of the values obtained for the concentration and temperature of the electrons.*

During rapid expansion of a plasma, such as occurs (for example) when there is supersonic flow into a vacuum, the result is a highly nonequilibrium state of the gas. Characteristic features of this state include a difference between the temperatures of the electrons and the heavy particles ($T_e > T$), increased electron concentration N_e relative to equilibrium and an excess concentration of atoms in the higher excited states. Under certain conditions, there may be inverse population of the excited states, which can be of definite practical interest. /610

The above phenomena have been studied theoretically and experimentally in [1-6]. The present paper contains a calculation of the plasma parameters, including a calculation of the population levels, for flow in a spherical source. We selected hydrogen as the working substance, because the values of the parameters entering into the calculation are relatively well-known.

[†] Leningrad State Pedagogical Institute imeni A.I. Gertsen.

* Numbers in the margin indicate pagination in the foreign text.

Basic Assumptions

We will assume that the hydrogen plasma on the original spherical surface consists of electrons, ions and atoms. At the initial parameters chosen (see below) the initial degree of dissociation is high, although the formation of molecules in the flow can be disregarded inasmuch as the recombination rate is low relative to the expansion rate. The equilibrium concentration of negative hydrogen ions under the conditions of interest to us here does not exceed 10^{-5} [7].

We will consider flow following the disturbance of ionization equilibrium, assuming that the velocity of all components is equal and that the plasma is quasineutral.

We will furthermore consider that the critical elementary process is collision-radiation recombination, which is a consequence (1) of the recombination of electrons into upper excited levels in triple collisions and (2) of cascade transition to the ground level (see for example [8]). For determining the kinetics of the populations of hydrogen levels during motion of the plasma, it is necessary to solve jointly a system of differential equations for population balance and equations of hydrodynamics.

/611

However, when the concentration of excited atoms $N_k (k \neq 1)$ satisfies the conditions

$$N_k \ll N_1, \quad N_k \ll N_e, \quad (k \neq 1), \quad (1)$$

it is possible to have a quasistationary solution to the system of equations for population balance. It results that the relaxation time of the $k \neq 1$ level is significantly less than the relaxation time of the ground level or of the free electrons. The population of the $k \neq 1$ level then changes extremely slowly in comparison to the rate of formation and destruction of the systems at the level in question. The differential equations for population balance are replaced by a system of algebraic equations which make it possible to determine $N_k (k \neq 1)$ from N_e and T_e , which are known from the hydrodynamic calculation. Then, in addition to condition (1), the following condition must be satisfied:

$$\tau_k \ll \tau_g \quad (2)$$

where τ_k is the relaxation time of the $k \neq 1$ level and τ_g is the characteristic hydrodynamic time.

The area of parameter changes covered below satisfies conditions (1) and (2). At considerable distances from the pole of the

source, with $T_e \lesssim 500^\circ\text{K}$, their destruction takes place, and the quasistationary approximation makes it possible to obtain only semi-quantitative results.

System of Equations

The system of hydrodynamic equations, including the equations of conservation of mass, momentum and energy as well as the equations of state, has the form

$$\frac{d}{dr}(N_e u r^2) = -\alpha N_e^2 r^2, \quad (3)$$

$$\frac{d}{dr}(N_a u r^2) = \alpha N_e^2 r^2, \quad (4)$$

$$dp_e/dr = -e N_e E, \quad (5)$$

$$m_a(N_e + N_a)u \frac{du}{dr} = -\frac{d(p_i + p_a)}{dr} + e N_e E, \quad (6)$$

$$\frac{d}{dr}\left(N_e u r^2 \frac{5}{2} k T_e\right) = Q r^2 - I \frac{d}{dr}(N_e u r^2) - N_e u r^2 e E, \quad (7)$$

$$\frac{d}{dr}\left[N_e u r^2 \left(\frac{5}{2} k T + \frac{m_a u^2}{2}\right) + N_a u r^2 \left(\frac{5}{2} k T + \frac{m_a u^2}{2}\right)\right] = -Q r^2 + N_e u r^2 e E, \quad (8)$$

$$p_e = N_e k T_e, \quad (9)$$

$$p_i = N_e k T, \quad (10)$$

$$p_a = N_a k T, \quad (11)$$

$$Q = \frac{N_e^2 e^4}{(4\pi e)^2 m_a} \left(\frac{8\pi m_e}{k T_e}\right)^{1/2} \left(\frac{T}{T_e} - 1\right) \ln \left[\frac{72\pi^2 (k T_e)^{3/2}}{N_e e^6} \right].$$

In the above, p = pressure, N = concentration, u = velocity of directed motion, m = particle mass, T = temperature, r = distance from the source pole. The subscripts e , i and a represent electrons, ions and atoms, respectively. The values of the recombination coefficient $\alpha(N_e, T_e)$ are taken from [8].

The rate of energy transmission from the electrons to the heavy particles in elastic collisions Q at a certain degree of ionization are wholly determined by the electron-ion collisions. The value I^* /612 represents that part of the recombination energy which returns to electron gas upon deactivation of the excited states by electron collisions of the second order. A portion $I - I^*$ of the recombination energy is radiated in spectral lines. A certain part of this energy can also be converted into thermal energy of electrons if a part of the radiation is resorbed in the plasma. Resorption of the resonance radiation plays a critical role in the energy relationship.

In order to show the effect of resorption on the plasma parameters, let us examine three cases:

(1) A plasma which is optically thin for all forms of radiation [9]:

$$I^* = 3.1 \cdot 10^{-4} N_e^0 T_e^0 I,$$

(2) A plasma which is optically thin for all forms of radiation except the L_α line:

$$(I_{\alpha} = 10.15 \text{ eV}).$$

(3) A plasma which is optically thick for all forms of radiation:

$$I^* = I. \quad (I = 13.53 \text{ eV}).$$

Plasma Parameters

With the aid of some simple transformations, the system of equations in (3-11) was converted to a system of five dimensionless first-order differential equations, suitable for machine calculation. Numerical integration was performed on the "Ural-1" computer using the Runge-Kutta variable-pitch method. The following raw data were selected: $r_0 = 1 \text{ cm}$, $N_{e0} = N_{i0} = 10^{15} \text{ cm}^{-3}$, $N_{a0} = 10^{17} \text{ cm}^{-3}$, $T_{e0} = 5 \cdot 10^3 \text{ }^\circ\text{K}$, $T_0 = 4.5 \cdot 10^3 \text{ }^\circ\text{K}$, $u_0 = 10^6 \text{ cm/sec}$.

The initial values of T_e and T_i were chosen from the balance conditions

$$Q = I^* \alpha N_e^2.$$

Figure 1 shows the results of calculating T_e , T_i , N_e , N_a and α for flow regions $1 \leq r \leq 10 \text{ cm}$. The values of the parameters for the optically thick plasma (Case 3) are practically the same as those for a plasma which is optically thick only in the L_α line (Case 2). A comparison of the findings in Cases 1, 2 and 3 shows that resorption of radiation leads to an increase in T_e , N_e and the degree of ionization, as well as a decrease in the recombination coefficient. The latter is explained by the strong temperature-dependence $\alpha \sim T^{-9/2}$. The recombination coefficient changes slightly at large r . The sharp increase in α in the vicinity of the initial surface is related to the assumed value of the electron temperature T_e , which (generally speaking) is determined by the previous history of the flow. With a real expansion of the plasma, we must expect smaller gradients α at $r \sim 1 \text{ cm}$.

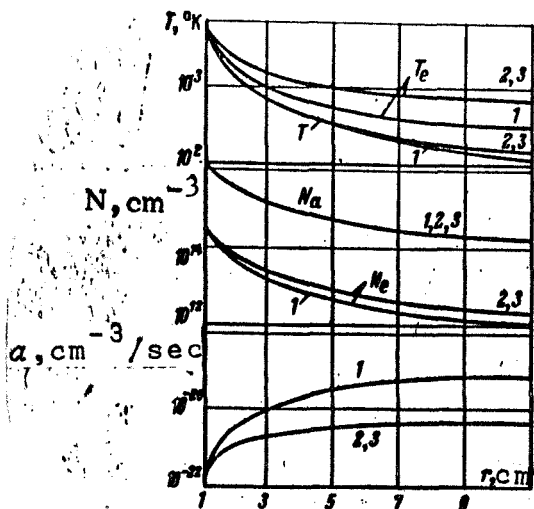


Fig. 1. Recombination Coefficient, Temperature, and Particle Concentration in a Supersonic Source:

- (1) Plasma Optically Thin for All Forms of Radiation;
- (2) Plasma Optically Thin for All Forms of Radiation Except the L_α Line;
- (3) Plasma Optically Thick for All Forms of Radiation.

population of the second level increases to 10^{12} - 10^{13} cm^{-3} (see below). Hence, there is no question of inversion with respect to the second level in a weakly-ionized plasma with significant optical thickness. Resorption of L_α is practically nonexistent in the populations N_3 and N_4 , if T_e does not exceed 2000 and 4000°K, respectively. Beginning with $r \sim 3$ cm, there is inversion for vapor of the levels 4-3, and beginning with $r \sim 7$ cm, for vapor of levels 5-3.

Coefficient of Amplification of Radiation and the Effect of Resorption

The coefficient of amplification of radiation with inverse population levels is calculated by the formula

$$K_{kl} = \frac{1}{4\pi^2} s(\lambda) \lambda_{kl}^2 g_k A_{kl} \left(\frac{N_k}{g_k} - \frac{N_l}{g_l} \right) \quad (12)$$

For the Doppler curve, $s(\lambda) = \sqrt{\pi \ln 2} \lambda^2 / c \Delta \lambda$.

For the Stark curve, $s(\lambda) = \lambda^2 / \pi c \Delta \lambda$, where $s(\lambda)$ is the coefficient of the shape of the spectral line and A_{kl} is the probability of a spontaneous transition.

Concentration of Excited Atoms

The concentrations of hydrogen atoms at various excited states are calculated as functions of T_e and N_e in [8, 10, 11]. Figures 2 and 3 show the populations of N_k for a series of low excited states, relative to the statistical weights g_k calculated with the data in [8] and the values we obtained for T_e and N_e for various r .

The higher excited states ($k > 5-6$) are in equilibrium with one another and with the electron continuum. In the case of an optically thin plasma, almost the entire flow field is characterized by inverse population of the upper levels relative to the second level (and beginning with $r \sim 3-4$ cm, relative to the third level as well). For a plasma which is optically thick in the L_α line, the

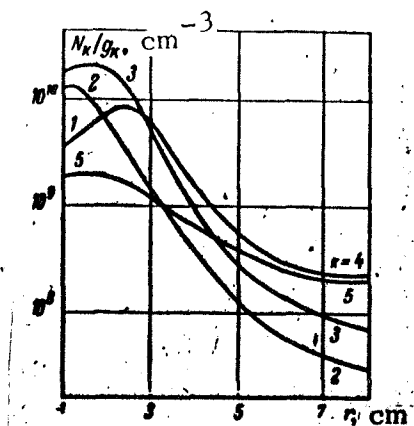


Fig. 2. Population of Levels in a Source Which Is Optically Thin for All Forms of Radiation.

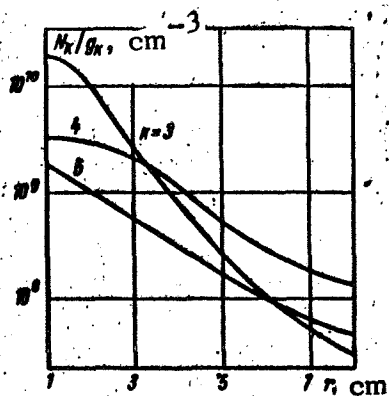


Fig. 3. Population of Levels in a Source Which Is Optically Thin for All Forms of Radiation Except the L_α Line.

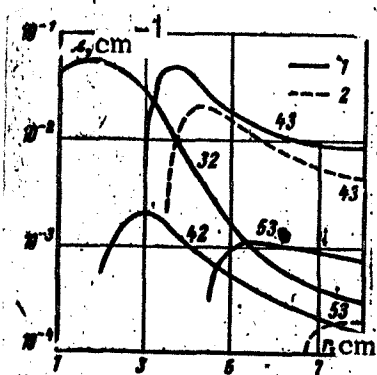


Fig. 4. Change in Coefficient of Amplification along the Radius of the Source.

For the source in question, the halfwidth of the lines $\Delta\lambda$ is within the limits from 0.2 to 2.5 Å, while the curve of the H_α line is determined practically completely by the Doppler effect; for transitions from the fourth and fifth levels at $r \lesssim 2-3$ cm, Stark expansion predominates.

The coefficients of amplification for an optically thin plasma and a plasma which resorbs only the L_α line are shown in Figure 4.

For transitions 4-3 ($\lambda = 1.88 \mu$) and 3-4 ($\lambda = 0.65 \mu$) in the case of an optically thin plasma, the coefficient of amplification is rather high. Even in the case of a radiant layer 1-3 cm thick, the amplification reaches a level of 3%, sufficient for compensation of final losses in the resonator. Considerable amplification is also achieved for the transition 4-2 (H_β), while in a plasma which is optically thick for L_α it is also obtained for the transition 4-3.

Resorption of radiation produced by transitions at the second level has little effect on the values of N_e and T_e , but can reduce inversion markedly. /614

Let us estimate the optical thickness of the source plasma in the L_α and H_α lines. Significant resorption of lines with a Doppler curve in a layer of thickness x occurs under the condition that

$$k(L_\alpha) = 6 \cdot 10^{-11} T_e^{-1/2} N_2 x > 1, \quad k(H_\alpha) = 5 \cdot 10^{-11} T_e^{-1/2} N_2 x > 1.$$

Assuming that the radiation leaves the source in a radial direction, for a point with the coordinate r we will assume the conditions

$$k(L_\alpha) \approx 10^{-13} N_1(r) r > 1, \quad k(H_\alpha) \approx 10^{-12} N_2(r) r > 1.$$

Numerical analysis will show that over the entire flow field $k(L_\alpha) \gg 1$, the resonance radiation is completely retained and the optically thin plasma is not realized. The population rate of the second level can be estimated by the formula

$$dN_2/dt = \alpha N_e^2 - R N_e N_2$$

(the values of the coefficient R are given in [8]). Even at a distance on the order of 0.1 cm from the original surface, N_2 reaches 10^{13} cm^{-3} and decreases further quite slowly. Hence $k(H_\alpha) > 1$, the H_α line is resorbed in a considerable part of the source, and the population N_2 can increase noticeably.

A more precise determination of the concentrations of excited atoms, inversion and the coefficients of amplification requires solution of the problem of radiation transfer in the source.

References

1. Gudzenko, A.I. and L.A. Shelepin: Doklady Akad. Nauk SSSR, Vol. 160, p. 1296, 1965.
2. Hertzberg, A. and J. Hurle: Phys. Fluids, Vol. 8, p. 1601, 1965.
3. Basov, N.G., A. N. Orayevskiy and V.A. Shcheglov: Zh. Tekh. Fiz., Vol. 37, p. 339, 1967.
4. Gudzenko, L.I., S. S. Filippov and L.A. Shelepin: Zh. Eksp. i Teor. Fiz., Vol. 51, p. 1115, 1966.
5. Gol'dfarb, V.M. et al.: Optika i Spektroskopiya, Vol. 20, p. 1085, 1966.
6. Gol'dfarb, V.M. and G.A. Luk'yanov: Zh. Tekh. Fiz., Vol. 38, No. 10, 1969.
7. Gross, R.A. and K.L. Eisen: Ionnyye, Plazmennyye i Dugovyye Raketnyye Dvigateli (Ion, Plasma and Arc Rocket Engines). Gosizdat, 1961.
8. Bates, D.R., A. E. Kingston and R.W.P. McWhirter: Proc. Roy. Soc., Vol. A267, p. 297, 1962; Vol. A270, p. 152, 1962.
9. Kuznetsov, I.M. and Iu.P. Rayzer: PMTF, No. 4, p. 10, 1965.
10. Drawin, H.W.: Ann. Physik, Vol. 14, p. 262, 1964.
11. Gordiets, B.F., L. I. Gudzenko and L.A. Shelepin: Preprint No. 29, Optical Laboratory of FIAN, 1967.

Translated for the National Aeronautics and Space Administration by:
Aztec School of Languages, Inc.,
Research Translation Division (90)
Maynard, Massachusetts and McLean, Virginia
NASW-1692